Permutations and Combinations

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. A dentist conducts a team to take surveys of people in his locality about using toothpaste. A survey team has some persons and the survey team owner makes a team out of total persons available at that time. If he has a group of 9 persons available at that time out of which 5 are men and 4 are women.

(A) In the commitee if it is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?
(B) If P(2n-1, n): P(2n + 1, n − 1) = 22:7, find n.
(C) From a team of 6 students, in how many ways can we choose a captain and vice-captain assuming one person can not hold more than one position?

Ans. (A) There are 9 seats, out of which 4 are at even places and rest are at odd places.
Thus, there ae 4 even places.
So, 4 women can be seated in 4 even places in 4! ways.
In rest of the places, five men can be placed in 5! ways.
Hence, required number of ways = 4! x 5!
= 24 x 120
= 2880
(B) We have,

$$^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22 : 7$$

$$\frac{\frac{(2n-1)!}{(2n-1-n)!}}{\frac{(2n+1)}{(2n+1-n+1)!}} = \frac{27}{7}$$

$$\frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)!}{(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)(n+1)n(n-1)!}{(2n+1)(2n)(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)n}{(2n+1)(2n)} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$$

$$7(n^2 + 3n + 2) = 44\,(2n + 1)$$

$$7n^2 + 21n + 14 = 88n + 44$$

$$7n^2 - 67n - 30 = 0$$

$$7n^2 - 70n + 3n - 30 = 0$$

$$7n(n - 10) + 3(n - 10) = 0$$

$$\Rightarrow \quad (n - 10)\,(7n + 3) = 0$$

$$\Rightarrow \quad n = 10, \quad \text{or} \quad n = \frac{-3}{7}$$

$$\therefore \quad n = 10$$

[$\because$ n can't be negative and fraction]

(C) From a team of 6 students, two students are to be chosen in such a way that one student will hold only one position. Here, the no. of ways of choosing a captain and vice-captain is the permutation of 6 different things taken 2 at a time.

So, $^6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 30$

2. Sumit works at a book shop. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.

(A) In how many ways can he select either a History or a maths book?

(a) 10

(b) 8

(c) 20

(d) 60

(B) If he selects 2 History books, 1 Math book and 1 Science book to arrange them, then find the number of ways in which selection can be made.

(a) 200

(b) 220

(c) 240

(d) 260

(C) Find the number of ways, if the books of the same subject are put together.

(a) 4! .2! .3!

(b) 2.3.2.5!

(c) 5! .2 .4!

(d) 3! .5 .4!

(D) If we are given the number of arrangement of books are 5P2 x $3P_1$ x $4P_1$, then the arrangement is in the manner:

(a) 2 History books, 2 Maths books, 3 Science books respectively.

(b) 2 History books, 3 Maths books, 2 Science books respectively.

(c) 3 History books, 2 Maths books, 2 Science books respectively.

(d) None of these

(E) In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together?

(a) 41472

(b) 42000
(c) 30000
(d) 50208

Ans. (A) (b) 8
Explanation: A History book can be selected in 5 ways and a Math book can be selected in 3 ways.
Required number of ways = 5 + 3 = 8
(B) (c) 240
Explanation: Now, 2 History books can be chosen in $^4P_1$ ways, 1 Maths book can be chosen in $^3P_1$ ways and 1 Science book can be chosen in $^4P_1$ ways.
.. Required number of ways = $^5P_2$ x $^3P_1$ x $4P_1$
= 240
(C) (d) 3!.51.4!
Explanation: Number of ways of arranging History books = 5!
Number of ways of arranging Maths books = 3!
Number of ways of arranging Science books = 4!
.. Required number of ways if the books of same subject are put together 3! .5!.4!
(D) (d) None of these
Explanation: The number of arrangements of books $^5P_2$ x 3P1 x $^4P_1$ represents the arrangement of 2 History books, 1 Maths book and 1 Science book respectively.
(E) (a) 41472
Explanation: First we take books of a particular subject as one unit.
Thus, there are 4 units which can be arranged in 4! = 24 ways.
Now in each of arrangements.
Mathematics books can be arranged in 3! ways.
History books in 4! ways.
Chemistry books in 3! ways.
And biology books in 2! ways.
Thus, the total number of ways = 4! x 3! x 4!
x 3! x 2!
24 x 6 x 24 x 6 x 2.
= 41472

3. Riya and her 5 friends went for a trip to Shimla. They stayed in a hotel. There were 4 vacant rooms A, B, C and D. Out of these 4 vacant rooms, two rooms A and B were double share rooms and two rooms C and D can contain one person each.

(A) The number of ways in which in which room A can be filled is:

(a) 10

(b) 15

(c) 20

(d) 25

(B) If room A and B are already filled each, then the number of ways in which room C and be filled is:

(a) 2

(b) 4

(c) 6'

(d) 8

(C) The total number of ways of accommodating Riya and her friends in these 4 vacant rooms is:

(a) 150

(b) 160

(c) 170

(d) 180

(D) If room A is filled with 2 persons, then the number in which rooms C and D can be filled is:

(a) 4

(b) 12

(c) 8

(d) 10

(E) The number of ways in which 10 digit numbers can be written using the digits 1 and 2 is:

(a) 210

(b) $^{10}C_2$

(c) 10!

(d) $^{10}C_1+^{9}C_2$

Ans. (A) (b) 15

Explanation: Total members = 6

Room A is a double shared room.

:- The number of ways in which room A can be filled = $^{6}C_2$ = 15

(B) (a) 2

Explanation: Now, rooms A and B can be filled with 2 members each and room C can be filled with 1 person.

.. Required number of ways = $^{2}C_1$ = 2

(C) (d) 180

Explanation: Required number of ways

= 15 x 6x2x1 = 180

(D) (b) 12

Explanation: As, room A is filled with 2 persons

Now, the remaining persons = 4

Given that room C and D can occupy 1 person each.

.. The number of ways in which rooms C and D can be filled = $^{4}C_1$ x $^{3}C_1$ = 12

(E) (a) 2^{10}

Explanation: Given digits are 1 and 2. Here, each place can be filled in two ways either with 1 or 2 and every place has two chances.

Therefore, the number of ways 10 digit numbers can be written using the digits 1 and 2 is 2^{10}

4. Two friends Swati and Komal are playing cards. Swati asks Komal to choose any four cards from a pack of 52 cards. Now, based on this answer the following:

(A) In how many ways can Komal select 4 cards from same suit and she select all 4 cards from different suits?

(B) In how many ways can Komal select 4 cards of different suit?

(C) In how many ways can she select 4 face cards from all face cards?

Ans. (A) Komal can select 4 cards from same suit either 4 hearts or 4 diamonds or 4 spades or 4 clubs.

i.e., ${}^{13}C_4+{}^{13}C_4+{}^{13}C_4+ {}^{13}C_1 = 4 \times {}^{13}C_4$

(B) She can select 4 cards from different suits as

1 heart, 1 diamond, 1 spade and 1 club

i.e., ${}^{13}C_1 \times {}^{13}C_1 \times {}^{13}{}_1, \times {}^{13}C_1$

$= 13 \times 13 \times 13 \times 13 = (13)^4$

(C) In a pack of 52 cards, there are 12 face cards and 40 non-face cards.

.. Number of ways of selecting 4 face cards from 12 face cards